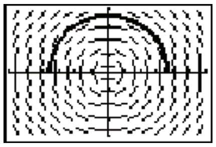
40. Graph (f)



Section 6.2

Exercises 6.2

78. (a) $d/dx\,[F(x) + C]$ should equal $f(x)$.

(b) The slope field should help you visualize the solution curve $y = F(x)$.

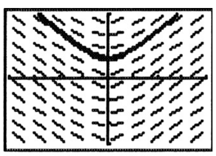
(c) The graph of $y_1 = F(x)$ and $y_2 = \int_0^x f(t)\,dt$ should differ only by a vertical shift, C.

(d) A table of values for $y_1 - y_2$ should show C.

(e) The graph of NDER of $F(x)$ and $f(x)$ should be the same.

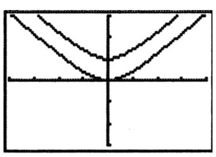
(f) a) $\frac{d}{dx}\sqrt{x^2 + 1} + C) = \frac{x}{\sqrt{x^2+1}}$

b)



[−4, 4] by [−3, 3]

c)

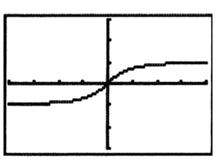


[−4, 4] by [−3, 3]

d)

x	$y_1 - y_2$
0	1
1	1
2	1
3	1
4	1

e)

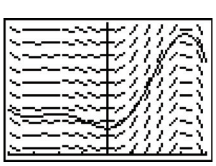


[−4, 4] by [−3, 3]

Section 6.3

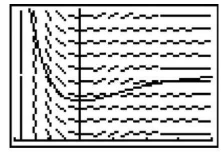
Exercises 6.3

11. $-(x + 2)\cos x + \sin x + 4$



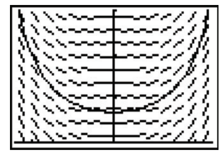
[–4, 4] by [0, 10]

12. $y = -2xe^{-x} - 2e^{-x} + 5$



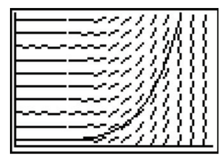
[–2, 4] by [0, 10]

13. $u = x\tan x + \ln|\cos x| + 1$



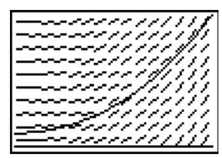
[–1.2, 1.2] by [0, 3]

14. $z = \frac{x^4}{4}\ln x - \frac{x^4}{16} + \frac{81}{16}$



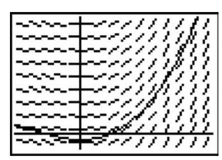
[0, 5] by [0, 100]

15. $y = \frac{2x}{3}(x - 1)^{3/2} - \frac{4}{15}(x - 1)^{5/2} + 2$



[1, 5] by [0, 20]

16. $y = \frac{4x}{3}(x + 2)^{3/2} - \frac{8}{15}(x + 2)^{5/2} + \frac{28}{15}$

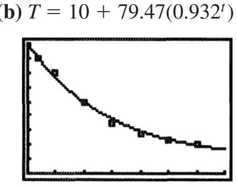


[–2, 4] by [–3, 25]

Section 6.4

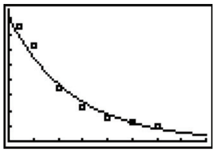
Exercises 6.4

33. (b) $T = 10 + 79.47(0.932^t)$



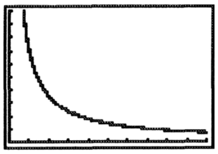
[0, 35] by [0, 90]

34. (b) $T = 79.96 \times 0.9273^t$



[–0, 40] by [0, 86]

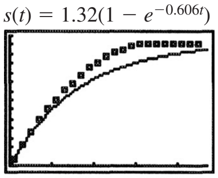
46. (b)



[0, 0.1] by [0, 100]

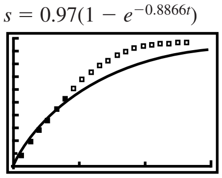
(c) $\ln 2 \approx 0.69$, so the doubling time is $0.69/r$ which is almost the same as the rules.

55. $s(t) = 1.32(1 - e^{-0.606t})$



[0, 4.7] by [0, 1.4]

56. $s = 0.97(1 - e^{-0.8866t})$



[0, 3] by [0, 1]

57. (a)

x	$\left(1 + \frac{1}{x}\right)^x$
10	2.5937
100	2.7048
1000	2.7169
10,000	2.7181
100,000	2.7183

$e \approx 2.7183$

(b) $r = 2$

x	$\left(1 + \frac{2}{x}\right)^x$
10	6.1917
100	7.2446
1000	7.3743
10,000	7.3876
100,000	7.3889

$e^2 \approx 7.389$

$r = 0.5$

x	$\left(1 + \frac{0.5}{x}\right)^x$
10	1.6289
100	1.6467
1000	1.6485
10,000	1.6487
100,000	1.6487

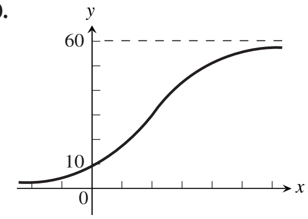
$e^{0.5} \approx 1.6487$

(c) As we compound more times the increment of time between compounding approaches 0. Continuous compounding is based on an instantaneous rate of change which is a limit of average rates as the increment in time approaches 0.

Section 6.5

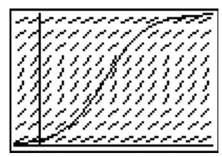
Quick Review 6.5

10.



Exercises 6.5

27. $P = \dfrac{200}{1 + 24e^{-1.2t}}$

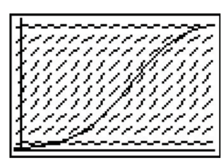


[–1, 7] by [0, 200]

28. $P = \dfrac{700}{1 + 69e^{-0.56t}}$

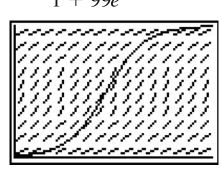
[–1, 15] by [0, 700]

29. $P = \dfrac{1200}{1 + 59e^{-0.24t}}$



[–1, 30] by [0, 1200]

30. $P = \dfrac{5000}{1 + 99e^{-0.05t}}$



[–1, 200] by [0, 5000]

35. Separate the variables and solve:

$$\frac{dP}{P(M-P)} = k\,dt$$

$$\frac{MdP}{P(M-P)} = Mk\,dt \qquad \text{Multiply by } M$$

$$\left(\frac{1}{P} + \frac{1}{M-P}\right)dP = Mk\,dt \qquad \text{Partial fractions}$$

$$\left(\frac{1}{P-M} - \frac{1}{P}\right)dP = -Mk\,dt \qquad \text{Multiply by } -1$$

$$\ln\left|\frac{P-M}{P}\right| = -Mk\,t + C$$

$$\left|1 - \frac{M}{P}\right| = e^{-Mk\,t} \cdot e^{C}$$

$$\frac{M}{P} = e^{-Mk\,t} \cdot A - 1 \qquad \text{Let } A = \pm e^{C}$$

$$\frac{P}{M} = \frac{1}{1 - Ae^{-Mk\,t}} \qquad \text{Reciprocate both sides}$$

$$P = \frac{M}{1 - Ae^{-Mk\,t}}$$

36. (a) Separate the variables and solve:

$$\frac{dP}{M-P} = k\,dt$$

$$-\ln|M-P| = kt + C$$

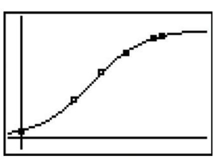
$$\ln|M-P| = -kt - C$$

$$M - P = Ae^{-kt} \qquad \text{Let } A = \pm e^{-C}$$

$$P = M - Ae^{-kt}$$

(d) This curve has no inflection point. If the initial population is greater than M, the curve is always concave up and approaches $y = M$ asymptotically from above. If the initial population is smaller than M, the curve is always concave down and approaches $y = M$ asymptotically from below.

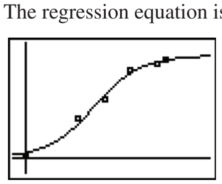
37. (a) The regression equation is $P = \dfrac{232739.9}{1 + 14.582e^{-0.101t}}$.



[–5, 70] by [–24000, 260000]

(b) Approximately 232,740 people.
(c) Sometime in the 60th year, that is, in 2010.
(d) $dP/dt = (4.352 \times 10^{-7})\,P(232739.9 - P)$

38. (a) The regression equation is $P = \dfrac{458791.8}{1 + 18.771e^{-0.113t}}$.



[–5, 70] by [–68000, 515000]

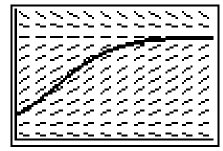
(b) Approximately 458,792 people.
(c) Sometime in the 60th year, that is, in 2010.
(d) $dP/dt = (2.4626 \times 10^{-7})\,P(458791.8 - P)$

45. (a) dP/dt has the same sign as $(M - P)(P - m)$.

(b) $P(t) = \dfrac{1200Ae^{11kt/12} + 100}{1 + Ae^{11kt/12}}$

(c) $P(t) = \dfrac{300(8e^{11kt/12} + 3)}{9 + 2e^{11kt/12}}$

(d)



[0, 75] by [0, 1500]

(e) $P(t) = \dfrac{AMe^{(M-m)kt/M} + M}{1 + Ae^{(M-m)kt/M}}$, where $A = \dfrac{P(0) - m}{M - P(0)}$

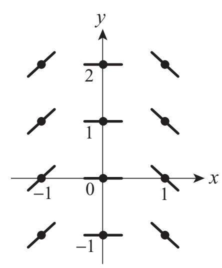
48. (a) This is true since $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3} = \dfrac{A(x-1)^2 + B(x-1) + C}{(x-1)^3}$.

(b) $A = 1$, $B = 5$, and $C = 9$.

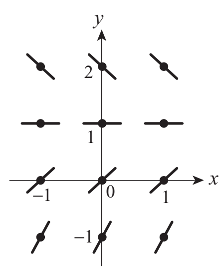
(c) $\ln|x-1| - \dfrac{5}{x-1} - \dfrac{9}{2(x-1)^2} + C$.

Review Exercises

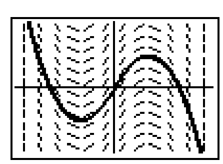
37.



38.



50.



[–10, 10] by [–10, 10]

62. Method 1—Compare graph of $y_1 = x^2 \ln x$ with

$y_2 = \text{NDER}\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right)$.

Method 2—Compare graph of $y_1 = \text{NINT}(x^2 \ln x)$

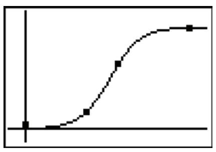
with $y_2 = \frac{x^3 \ln x}{3} - \frac{x^3}{9}$.

64. (a) $\frac{d}{dx}\int_0^x u(t)\,dt = u(x)$

$\frac{d}{dx}\int_3^x u(t)\,dt = u(x)$

(b) $C = \int_0^3 u(t)\,dt$

65. (a) The regression equation is $y = \dfrac{272286.4}{1 + 302.69e^{-0.2095t}}$. The graph is shown below.

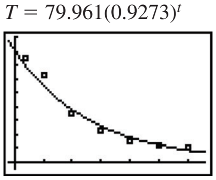


(b) 272, 286 people

(c) $dP/dt = 7.694 \times 10^{-7}\, P(272286.4 - P)$

(d) The carrying capacity drops to 267,312.6, which is below the actual 2003 population. The logistic regression is strongly affected by points at the extremes of the data, especially when there are so few data points being used. While the fit may be more dramatic for a small data set, the equation is not as reliable.

66. (a) $T = 79.961(0.9273)^t$



[–1, 33] by [–5, 90]

(b) About 9.2 sec

(c) About 79.96°C

68. (a) $dP/dt = k(600 - P)$. Separate the variables to obtain

$$\frac{dP}{600 - P} = k\,dt$$
$$\frac{dP}{P - 600} = -k\,dt$$
$$\ln|P - 600| = -kt + C_1$$
$$P - 600 = Ce^{-kt}$$
$$200 - 600 = Ce^0 \Rightarrow C = -400$$
$$P - 600 = -400e^{-kt}$$
$$P(t) = 600 - 400e^{-kt}$$

(b) $500 = 600 - 400e^{-k \cdot 2}$

$1/4 = e^{-2k}$

$k = \ln 2 \approx 0.693$

(c) $\lim_{t\to\infty}(600 - 400e^{-0.693t}) = 600$

69. (a) Separate the variables to obtain

$$\frac{dv}{v + 17} = -2dt$$
$$\ln|v + 17| = -2t + C_1$$
$$v + 17 = Ce^{-2t}$$
$$-47 + 17 = Ce^0 \Rightarrow C = -30$$
$$v + 17 = -30e^{-2t}$$
$$v = -30e^{-2t} - 17$$

(b) $\lim_{t\to\infty}(-30e^{-2t} - 17) = -17$ feet per second

(c) $-20 = -30e^{-2t} - 17$

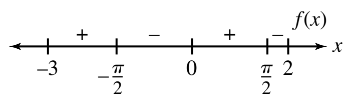
$t = \frac{\ln 10}{2} \approx 1.151$ seconds

CHAPTER 7

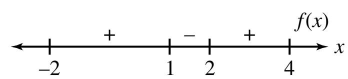
Section 7.1

Quick Review 7.1

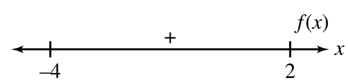
1. Changes sign at $-\frac{\pi}{2}, 0, \frac{\pi}{2}$



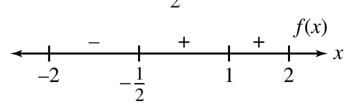
2. Changes sign at 1, 2



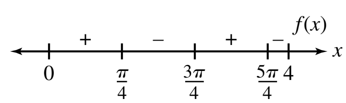
3. Always positive



4. Changes sign at $-\frac{1}{2}$

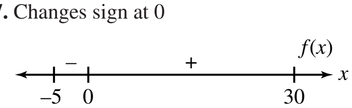


5. Changes sign at $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

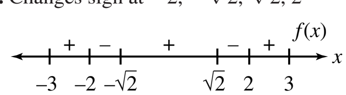


6. Always positive

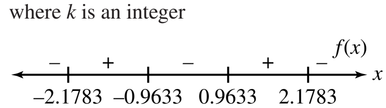
7. Changes sign at 0



8. Changes sign at $-2, -\sqrt{2}, \sqrt{2}, 2$



9. Changes sign at $0.9633 + k\pi$
$2.1783 + k\pi$
where k is an integer



10. Changes sign at $\frac{1}{3\pi}, \frac{1}{2\pi}$